LOYOLA COLLEGE (AUTONOMOUS) CHENNAI – 600 034

M.Sc. DEGREE EXAMINATION – **STATISTICS**

FIRST SEMESTER – **NOVEMBER 2024**

PST1MC04 – SAMPLING THEORY

Date: 15-11-2024 Dept. No. Max. : 100 Marks

Time: 01:00 pm-04:00 pm

	SECTION A – K1 (CO1)
	Answer ALL the questions **(5 x 1 = 5)**
1	**Define the following**
a)	Inclusion Indicators and Inclusion Probabilities.
b)	Midzuno Sampling design.
c)	Systematic Sampling.
d)	Hansen-Hurwitz Estimator.
e)	Ratio Estimator.
	SECTION A – K2 (CO1)
	Answer ALL the questions **(5 x 1 = 5)**
2	**Fill in the blanks**
a)	For any Sampling Design P(.), $V_P[I_i(s)] =$ _________________.
b)	An unbiased estimator of Y under random group method is ____________.
c)	In Linear Systematic Scheme, the constant **k** is known as _________________.
d)	The ratio estimator is a particular case of ______________________.
e)	The __________ technique is a method used in large-scale surveys to address the issue of non-response by re-contacting non-respondents.
	SECTION B – K3 (CO2)
	Answer any THREE of the following **(3 x 10 = 30)**
3	For any fixed size sampling design P(.), prove that (i) $\sum_{j=1}^{N} \pi_{ij} = (n-1)\,\pi_i$; $i = 1, 2, \dots, N;\ j \neq i$ (ii) $\sum_{j=1}^{N} (\pi_i \pi_j - \pi_{ij}) = \pi_i (1 - \pi_i);\ i = 1, 2, \dots, N;\ j \neq i.$
4	Prove that one can have more than one unbiased estimator for a given sampling design.
5	Explain Lahiri's Method and show that it is a Probability Proportion to Size (PPS) selection method.
6	Show that $\hat{Y}_{HHE}$ is unbiased for Y. Also derive the variance of Hansen-Hurwitz Estimator (HHE).
7	Describe Simmons Randomized Response Technique and derive the variance of estimator $\hat{\pi}_A$ when π_Y is unknown.

	SECTION C – K4 (CO3)
	Answer any TWO of the following **(2 x 12.5 = 25)**
8	a). Explain the design of Simple Random Sampling Without Replacement (SRSWOR) and describe its unit drawing mechanism. (4.5) b). In SRSWOR, show that $\Pi_i = \frac{n}{N}; i = 1, 2, \dots, N \quad and \quad \Pi_{ij} = \frac{n(n-1)}{N(N-1)}; i, j = 1, 2, \dots, N.$ (8)
9	Under Midzuno sampling Design, show that (i) the first order inclusion probability is $\Pi_i = \frac{N-n}{N-1} . \frac{X_i}{X} + \frac{n-1}{N-1}$, i=1,2…, N, and (ii)the second order inclusion probability is $\Pi_{ij} = \frac{(N-n)(n-1)}{(N-1)(N-2)} . \frac{X_i+X_j}{X} + \frac{(n-1)(n-2)}{(N-1)(N-2)}$, i ≠ j = 1, 2, …, N.
10	In Stratified Random Sampling, demonstrate that an Unbiased Estimate of Y is $\hat{Y}_{st} = \sum_{h=1}^{L} \hat{Y}_h$. Also, derive the formula for $\hat{Y}_{st}, \; V(\hat{Y}_{st})$ and $v(\hat{Y}_{st})$ under the design of (i) SRSWOR and (ii) PPSWR.
11	Derive the Hartley–Ross unbiased ratio type estimator for the population total ‘Y’.
	SECTION D – K5 (CO4)
	Answer any ONE of the following **(1 x 15 = 15)**
12	Explain the concept of Stratified Sampling and discuss Proportional Allocation within this framework. Additionally, derive the variance $V(\hat{Y}_{st})$ under proportional allocation.
13	Provide a detailed explanation of Warner’s Model and derive the estimated variance of $\hat{\Pi}_A$.
	SECTION E – K6 (CO5)
	Answer any ONE of the following **(1 x 20 = 20)**
14	a). Describe Hurwitz-Thomson (HT) Estimator and show that the estimator $\hat{Y}_{HT}$ is unbiased for Y under any sampling design P(.). (10) b). Verify if $\hat{Y}_{HT}$ is unbiased for ‘Y’ using (i) the definition of expectation and (ii) an expression involving inclusion indicators under the design $P(s) = \begin{cases} \frac{1}{7} \; if \; s = \{1, 2\} \\ \frac{3}{7} \; if \; s = \{2, 3, 4\} \\ \frac{3}{7} \; if \; s = \{3, 4, 5\} \\ 0, otherwise \end{cases}$ Given $Y_1 = 4$, $Y_2 = 3$, $Y_3 = 5$, $Y_4 = 2$, and $Y_5 = 7$. (10)
15	Explain Regression Estimation and derive an approximate expression for both the bias and mean squared error (MSE) of the estimator $\hat{Y}_{LR}$.

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